

Combinations of Functions

Let $f(x) = x^2 - 4$ and $g(x) = 3x + 1$

Sum:
$$\begin{aligned} f(x) + g(x) &= (f + g)(x) \\ (x^2 - 4) + (3x + 1) &= x^2 + 3x - 3 \end{aligned}$$

Difference:
$$\begin{aligned} f(x) - g(x) &= (f - g)(x) \\ (x^2 - 4) - (3x + 1) &= x^2 - 3x - 5 \end{aligned}$$

Product:
$$\begin{aligned} f(x) \cdot g(x) &= (fg)(x) \\ (x^2 - 4) \cdot (3x + 1) &= 3x^3 + x^2 - 12x - 4 \end{aligned}$$

Quotient:
$$\frac{f(x)}{g(x)} = \frac{x^2 - 4}{3x + 1} = \left(\frac{f}{g} \right)(x) \quad \text{for } g(x) \neq 0$$

The Domain of the Combination of two functions is the Intersection of the Domains of those two functions.

$$\text{Domain}_{f \pm g} = \text{Domain}_f \cap \text{Domain}_g$$

$$\text{Domain}_{f \cdot g} = \text{Domain}_f \cap \text{Domain}_g$$

$$\text{Domain}_{f/g} = \text{Domain}_f \cap \text{Domain}_g \cap \{x \mid g(x) \neq 0\}$$