

# The Difference Quotient: $\frac{f(x+h) - f(x)}{h}$

Find the Difference Quotient for  $f(x) = x^3$

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Scratch Work

$$f(x) = x^3$$

$$f(x+h) = (x+h)^3$$

$$= (x+h)(x+h)(x+h)$$

$$= (x^2 + 2xh + h^2)(x+h)$$

$$= x^3 + 2x^2h + xh^2 + x^2h + 2xh^2 + h^3$$

$$= x^3 + 2x^2h + 1x^2h + 1xh^2 + 2xh^2 + h^3$$

$$= x^3 + 3x^2h + 3xh^2 + h^3$$

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$$\frac{f(x+h) - f(x)}{h} = \frac{(x^3 + 3x^2h + 3xh^2 + h^3) - x^3}{h}$$

$$= \frac{(\cancel{x^3} + 3x^2h + 3xh^2 + h^3) - \cancel{x^3}}{h}$$

$$= \frac{3x^2h + 3xh^2 + h^3}{h}$$

$$\frac{f(x+h) - f(x)}{h} = 3x^2 + 3xh + h^2$$